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NASA Project Apollo Working Paper No. 1106

AN ANALYTICAL STUDY OF THE LANDING FOOTPRINT AVAILABLE  
DURING LEM LUNAR LANDING APPROACHES

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DURING LEM LUNAR LANDING APPROACHES

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## AN ANALYTICAL STUDY OF THE LANDING FOOTPRINT AVAILABLE DURING LEM LUNAR LANDING APPROACHES

### SUMMARY

The available landing area that can be reached by the additional expenditure of main propulsion fuel has been analyzed for typical LEM lunar landing trajectories. The footprints have been determined for specific altitudes of 15,000 feet, 10,000 feet, and 5,000 feet.

The landing footprints are superimposed upon a possible LEM window configuration as they would appear to the pilot during a typical approach. The application of the footprint information to aid pilot control procedure is also briefly discussed.

### INTRODUCTION

The design of a lunar descent trajectory, such as that required by the Apollo LEM, requires consideration of such factors as fuel expenditure, abort safety, landing approach velocities, et cetera, and the possible participation of the spacecraft crew in the control of the landing approach. If the crew is to provide a judgment factor and control the landing of the spacecraft, then the landing approach must allow for the pilot viewing the landing area with adequate time to assess the suitability of his approach trajectory and the suitability of the landing surface. Reference 1 presents an analysis of the tradeoffs that are possible between some of the more important factors during the portion of the landing approach where possible crew participation must be considered. From such information as reference 1, the landing approach trajectory can be shaped such that the landing area will be within the limits of a given window design and the approach velocities allow a reasonable time for assessing the landing surface without incurring prohibitive expenditures of fuel.

Although the work of reference 1 recognizes the value of the spacecraft crew assessing the landing area during the approach from several thousand feet of altitude, the analysis describes only the landing area that is available to the crew after the trajectory reaches one thousand feet. Depending upon lighting conditions, the crew may, however, decide upon a suitable landing position prior to reaching 1,000 feet altitude. It is necessary then for the crew to know the range capability of the spacecraft to be assured of choosing a site that can be reached within the fuel constraints of the mission.

To provide such landing area footprint information, an analytical analysis of the fuel cost for modifying the range of the LEM approach trajectory has been conducted by the Flight Dynamics Branch. The analysis was based upon landing approach trajectories which, after reaching a point of transition, maintain an approximately constant flight path angle and constant pitch and/or roll attitudes. Typical trajectories are chosen from reference 1 to provide the initial conditions for the analysis. The purpose of this report is to present the results obtained and to discuss the possible application to the LEM mission.

### ASSUMPTIONS

Because the details of how the landing approach will be guided have not been fully decided, the analysis has assumed a method of guidance that utilizes constant accelerations to maintain the desired flight path angle and to linearly decrease both horizontal and vertical velocities to zero as zero altitude is reached. While it is likely that the method of guidance that is implemented will differ from that assumed in many details, it is expected that the relative measure of fuel expenditure for lengthening or shortening the range traveled will be typical.

The analysis is based upon initial conditions derived from typical trajectories chosen from reference 1. Because transition altitude is one of the tradeoffs in trajectory design, the trajectories start from as high as 15,000 feet. An analysis is then made of the footprint available at 10,000 and 5,000 feet. The footprint in each case is applicable only if the nominal trajectory is flown to the altitude for which the footprint is determined.

The analysis of descents from a given altitude and velocity is based upon the following assumptions:

- (a) Constant mass
- (b) Constant acceleration of main engine for a given throttle setting
- (c) Flight path angle is constant
- (d) End conditions desired are zero velocity and zero altitude
- (e) A constant pitch angle is held

For the actual LEM descent from the transition-altitude, the mass will vary approximately 13 percent. Assumptions (c), (d), and (e) would be

valid in the actual descent providing the throttle is varied to maintain a constant acceleration as the mass changes. Because characteristic velocity is a function of only the applied acceleration (fuel, but not characteristic velocity, is a function of throttle variation), the velocity computed with the stated assumptions would be identical to the actual case.

The initial conditions for a given analysis are chosen from reference 1 and specify a certain altitude, horizontal velocity, and vertical velocity. These conditions in turn specify a nominal flight path angle. To utilize the above assumptions, it was necessary to assume that the vertical velocity could be instantaneously changed at the start of each trajectory to the value which, when coupled with the specified horizontal velocity, would produce the desired flight path angle. The characteristic velocity required to perform the maneuver, however, was added to the total characteristic velocity expenditure.

It was assumed that the main engine thrust could be adjusted to any desired value. Full throttle (throttle setting = 1.0) produced a thrust to weight ratio of 0.8 and incremental throttle settings produced a proportional thrust to weight ratio.

## SCOPE OF ANALYSIS

Four basic trajectories were chosen from reference 1 to analyze the landing footprint. These trajectories include two that are believed to be reasonably close to desirable lunar landing approach trajectories (flight path angles of  $12^\circ$  and  $14^\circ$ ). Also included are two trajectories with relatively steep flight path angles ( $33^\circ$  and  $60^\circ$ ) which were chosen to illustrate the effect of flight path angle upon the available footprint. Pertinent initial conditions of the trajectories analyzed are summarized in table I.

## RESULTS AND DISCUSSION

### Analysis of Available Footprint

General.— The results of the analyses of available footprints from each of the initial conditions specified in table I are presented in figures 2 through 10. The figures present plots of lunar surface range along and perpendicular to the initial flight path that may be obtained with specified penalties of characteristic velocity. Contours of equal value of characteristic velocity and also contours of equal value of throttle setting are shown on the figures. For the Type I trajectory,

it appears that for each 100 ft/sec of fuel penalty, the range may be extended about 9,000 feet if the change is initiated at 15,000 feet. At 10,000 feet (fig. 3) this has been reduced to approximately 7,500 feet per 100 ft/sec and at 5,000 feet (fig. 3) to only 5,000 feet additional range per 100 ft/sec. For the shallower flight path angle ( $\gamma = 12^\circ$ ) shown in figures 5 and 6, the shape of the range ( $\Delta V_c$  contours) is very similar to those for  $\gamma = 14^\circ$  (figs. 1 to 3), but the range sensitivity is slightly greater, more range per  $\Delta V_c$  penalty. The reason is primarily that the  $\gamma = 12^\circ$  trajectory reaches the 10,000 and 5,000 feet altitude position with slightly higher forward velocity than the  $\gamma = 14^\circ$  trajectory and this coupled with the shallower flight path angle results in a greater range sensitivity for  $\Delta V_c$  penalty. The comparison is not complete, however, without considering the characteristic velocity of the nominal trajectories. Reference 1 shows that the  $\gamma = 14^\circ$  trajectory nominally takes about 150 ft/sec more characteristic velocity than the  $\gamma = 12^\circ$  trajectory, but that this added 150 ft/sec results in a somewhat more favorable viewing angle of the landing area and also more time to view the landing area. The relative importance of the tradeoffs apparent between these two trajectories can be fully assessed only after the lighting conditions of the lunar surface are better understood and LEM window configuration is adopted.

The curves of figures 2 to 5 show that landing sites at shorter than nominal range may be reached with a saving of fuel. This results because the trajectories required to reach the shorter ranges are closer to a fuel optimum trajectory. It should be remembered that the original reasons for departing from a fuel optimum trajectory during the landing approach included improving the visibility of the landing site and abort safety (avoiding high vertical velocities), et cetera. Therefore, an attempt to utilize the range shortening capabilities would tend to destroy the advantages that were gained by shaping the trajectory away from the fuel optimum. For this reason, the ability to appreciably shorten the range is not of primary interest.

The capability to alter the trajectory laterally is shown (figs. 2 to 6) to be less costly than downrange alteration. The footprint plots include only the area which can be reached by a  $\pm 50^\circ$  change in the flight direction to put a constraint on possible abort conditions.

Effect of flight path angle.— As the flight path angle during the landing approach is increased, the ability to modify the landing site is decreased. Figures 7 and 8 present the footprints associated with the steeper flight path angles of  $33^\circ$  and  $60^\circ$ , respectively. It may be

seen that the extension of range capability is a fraction (about  $\frac{1}{4}$  and  $\frac{1}{6}$ , respectively) of that for the corresponding altitudes and  $\gamma = 14^\circ$  trajectory. The importance of this reduction in footprint area would appear to be another tradeoff in the ultimate selection of an approach trajectory. The vehicle pitch attitude as a function of throttle setting for three flight path angles is shown in figure 9.

#### Application of Landing Footprint Information

Pilot display.— For the spacecraft pilot to utilize the landing footprint information, it would appear desirable that he be given a window reflected reticle which would indicate the nominal landing point as well as the range contours corresponding to the  $\Delta V_c$  that could be utilized for range alteration purposes. With this display, the pilot could evaluate the nominal landing point and if unsuitable could limit his assessment of other landing sites to those that are within the attainable footprint. The provision of such a display is a rather difficult problem considering the changes that occur as the approach progresses and the need to stabilize the display independent of spacecraft attitude deviations. Some appreciation for the appearance of footprint information to the pilot may be obtained by superimposing a typical footprint upon a sketch of the viewing limits of a possible LEM window configuration. Figure 10 presents such information for the 15,000-ft, 10,000-ft, and 5,000-ft footprints associated with trajectory I. The figure presents the contours for  $\Delta V_c$  penalties of 100 to 400 ft/sec. The figures show that because of the perspective afforded the pilot (due to the relatively shallow flight path angle), the footprints subtend a small viewing angle. Although the footprint available at 15,000 feet is larger than those available at the 10,000 and 5,000 altitude positions, the visual angle subtended by the footprint grows slightly as the landing site is approached. This occurs because the range decreases at a more rapid rate than the footprint decreases. The possibility exists that a single display could be applied throughout most of the landing approach, providing that the contours corresponding to the  $\Delta V_c$  allowed for range adjustment are chosen conservatively. The present analysis has not, however, been extended to include the possibility of making more than one adjustment in the landing approach trajectory. The footprints presented, therefore, are only applicable for the first trajectory modification. Unless a means of space stabilizing the display is devised, the validity of a displayed footprint is dependent upon accurate attitude control. If the pilot takes over and deviates from the prescribed nominal attitude, the correct footprint must be shifted relative to the window by the amount of the attitude deviation to give a correct indication.

## CONCLUDING REMARKS

The analysis of landing footprints has indicated the approximate landing area that can be reached from specified points of typical lunar landing approach trajectories. A suitable method of application of the information to aid the pilot in evaluating available landing area could be by a contour-reticle projection, but the problems of implementing such a device do not appear easily solvable. It is recommended that additional studies be conducted to investigate the procedures whereby the crew could utilize such information for both manual and automatic lunar landing approaches.

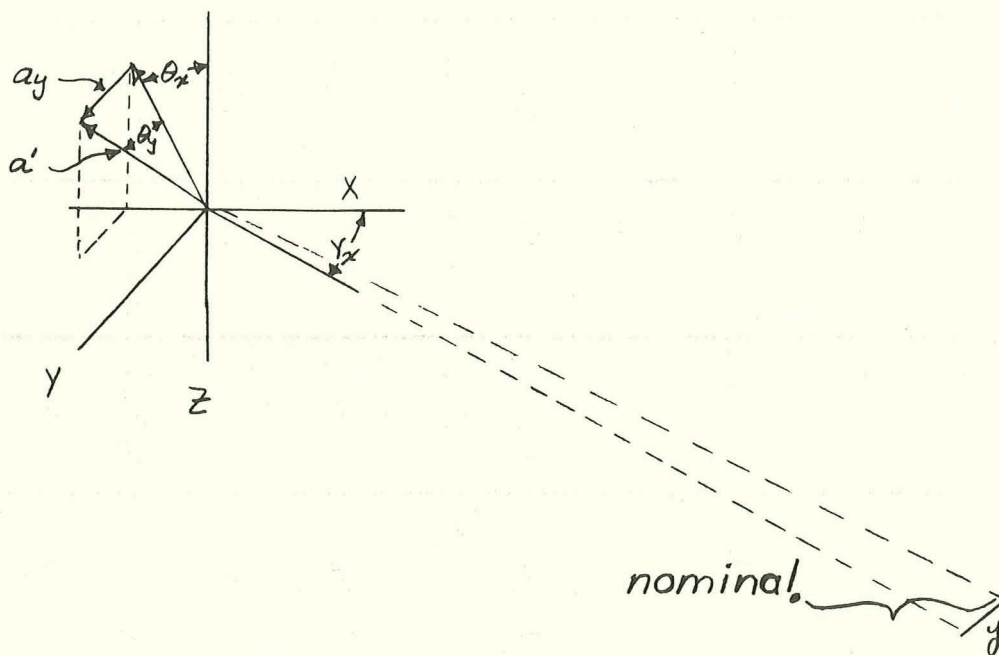
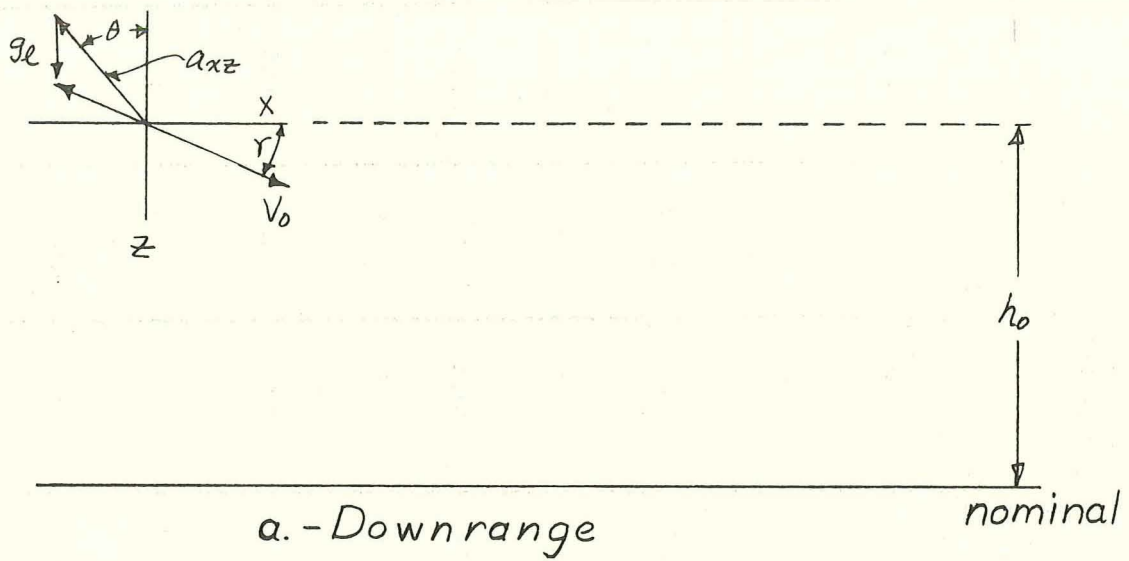
## REFERENCES

1. Bennett, Floyd, and Price, Thomas: "Study of Powered Descent Trajectories for Manned Lunar Landings." Project Apollo Working Paper No. 1084.

TABLE I.- INITIAL CONDITIONS FOR TRAJECTORIES ANALYZED

Trajectories	I			II		III		IV	
IC number . . . .	1	2	3	4	5	6	7	8	9
Nominal LEM throttle . . . . setting	.4	.4	.4	.5	.5	.5	.5	.5	.5
$A_{xz}$ (ft/sec <sup>2</sup> ) . . . .	8.89	8.89	8.89	10.9	10.9	10.9	10.9	10.9	10.9
Nominal pitch angle . . . . (deg)	40	40	40	50	50	33	33	10	10
$h_o$ (ft) . . . . .	15,000	10,000	5,000	10,000	5,000	10,000	5,000	10,000	50,000
$\gamma_o$ . . . . .	14.4	14.4	14.4	12	12	33	33	60	60
$\dot{x}_o$ (ft/sec) . . . . .	817	668	472	880	622	428	311	185	147
$v_o$ (ft/sec) . . . . .	844	-	-	900	-	510	-	370	-
$x_o$ (ft) . . . . .	-57,630	-37,700	-18,030	-47,000	-21,500	-15,700	-8,000	-7,000	-4,000
M (slugs) . . . . .	450	450	450	450	450	450	450	450	450

Note: IC numbers 2, 3, 5, 7, and 9 were obtained by flying the nominal trajectories of IC numbers 1, 4, 5, and 8 to the initial altitudes.



b. - Downrange plus Crossrange

Figure 1.- Coordinate system for calculation of footprint  $\Delta V$ .

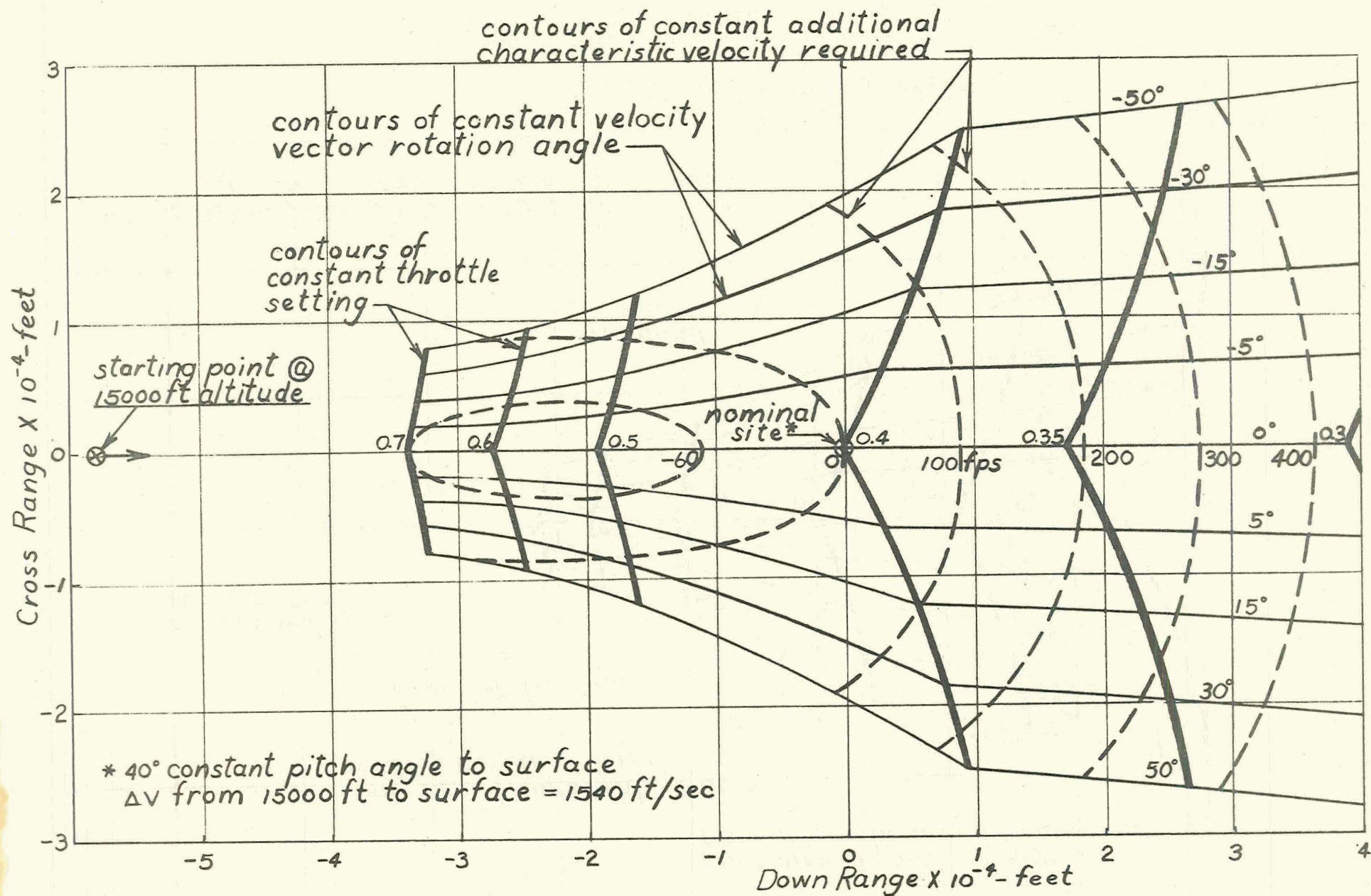


Figure 2.- Landing area footprint for corrections made at 15,000 feet altitude with velocity = 844 ft/sec and flight path angle of 14°.

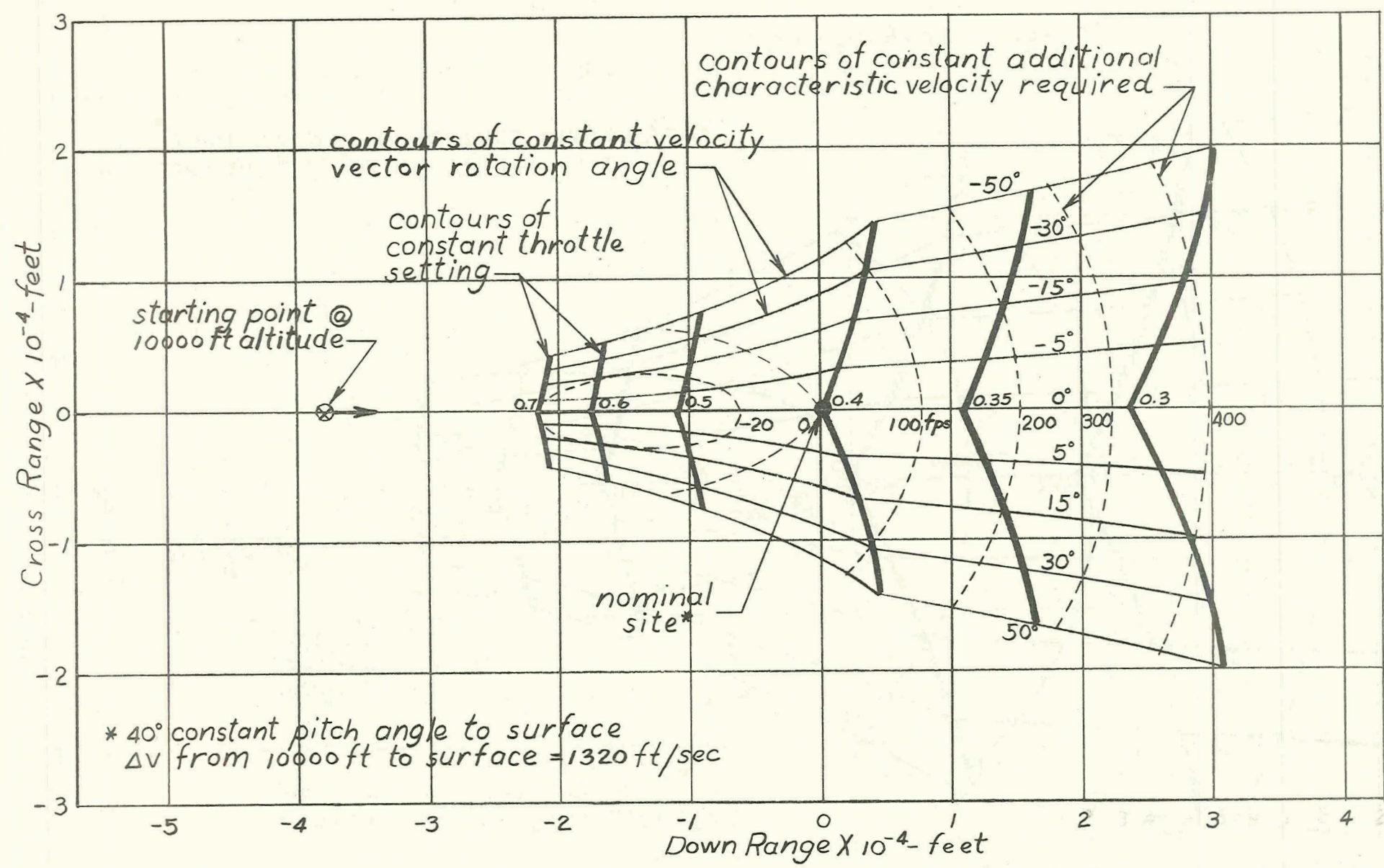


Figure 3.- Landing area footprint for corrections made at 10,000 feet altitude with velocity = 688 ft/sec and flight path angle of  $14^\circ$ .

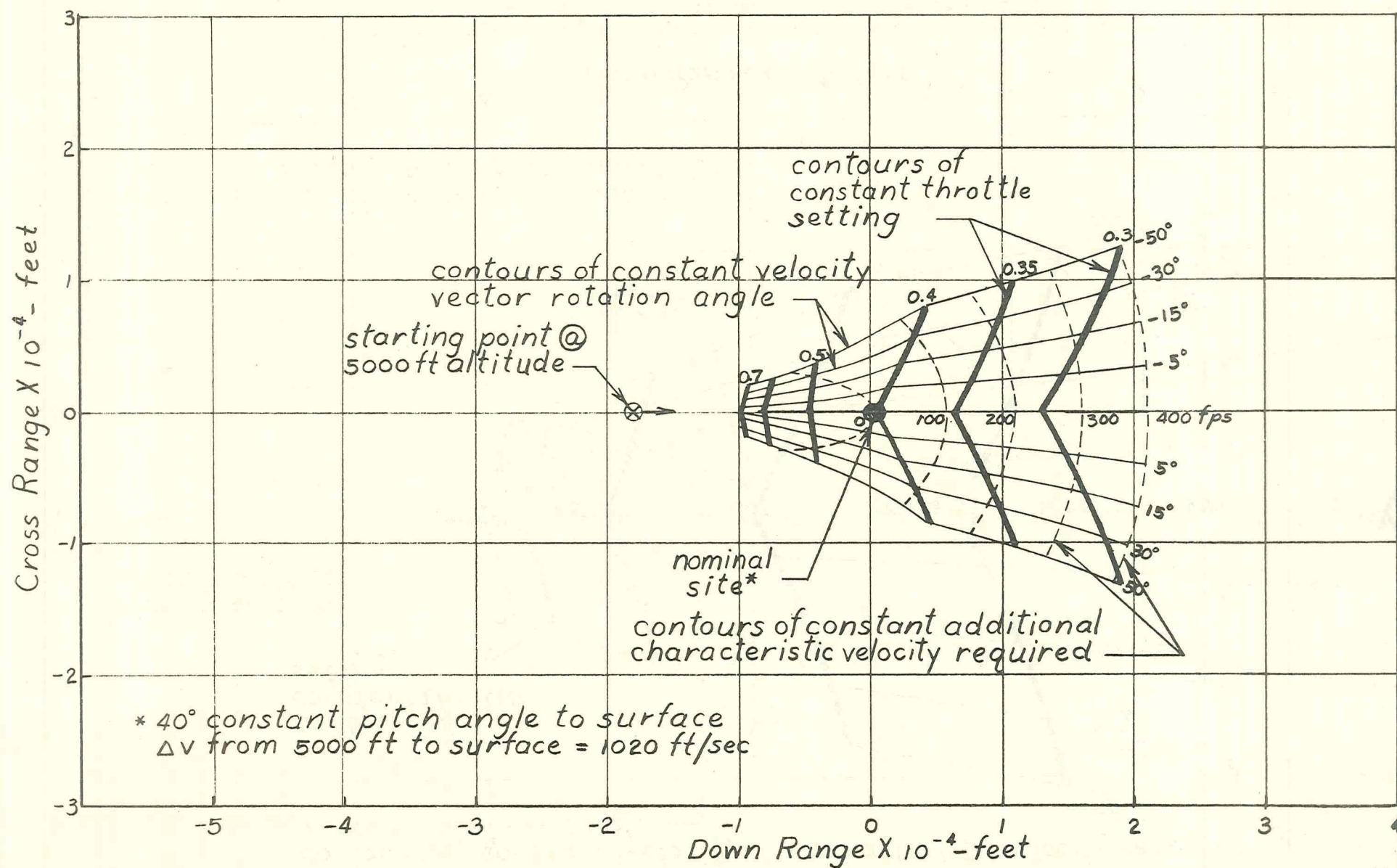


Figure 4.- Landing area footprint for corrections made at 5,000 feet altitude with velocity = 485 ft/sec and flight path angle of 14°.

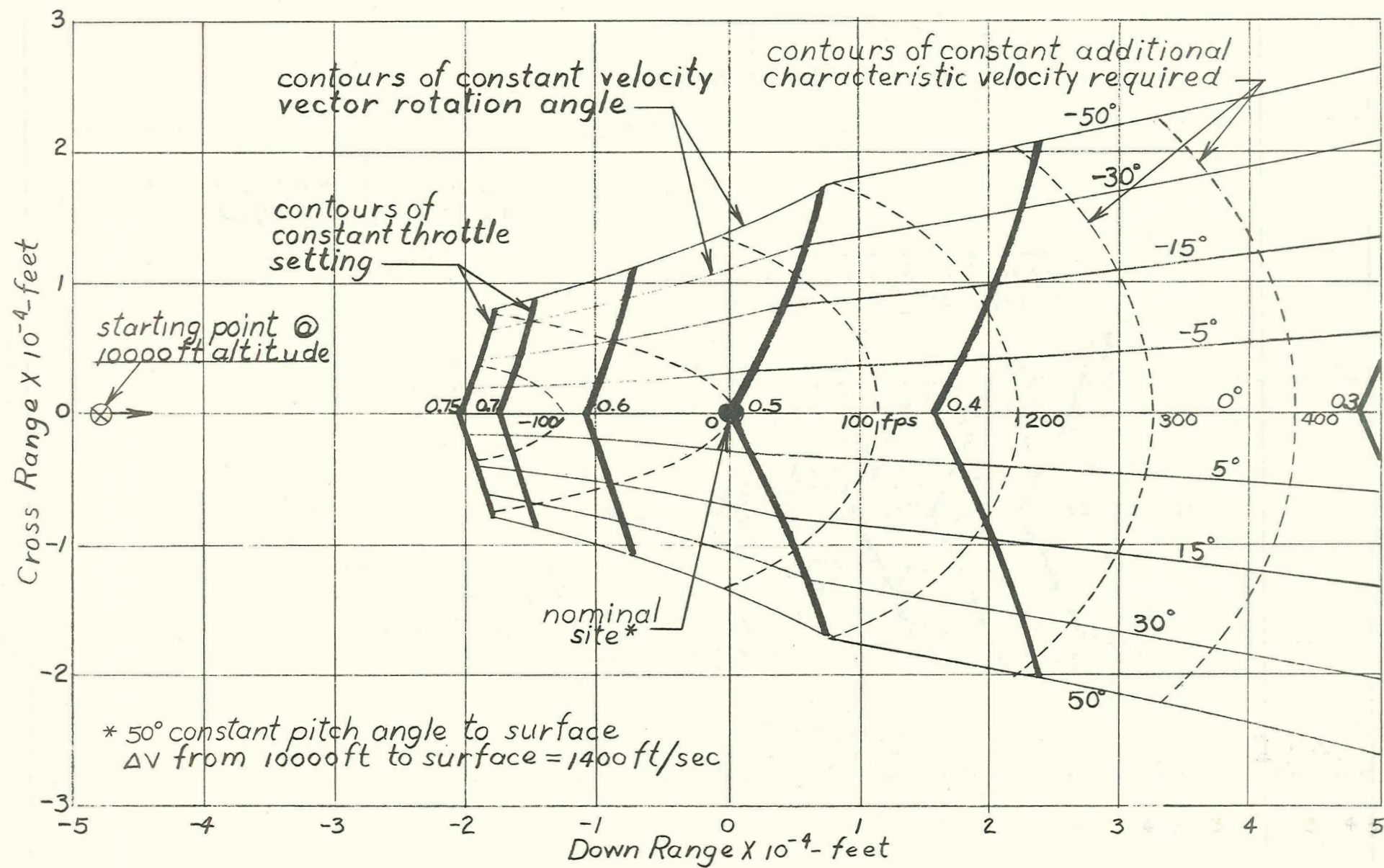


Figure 5.- Landing area footprint for corrections made at 10,000 feet altitude with velocity = 900 ft/sec and flight path angle of 12°.

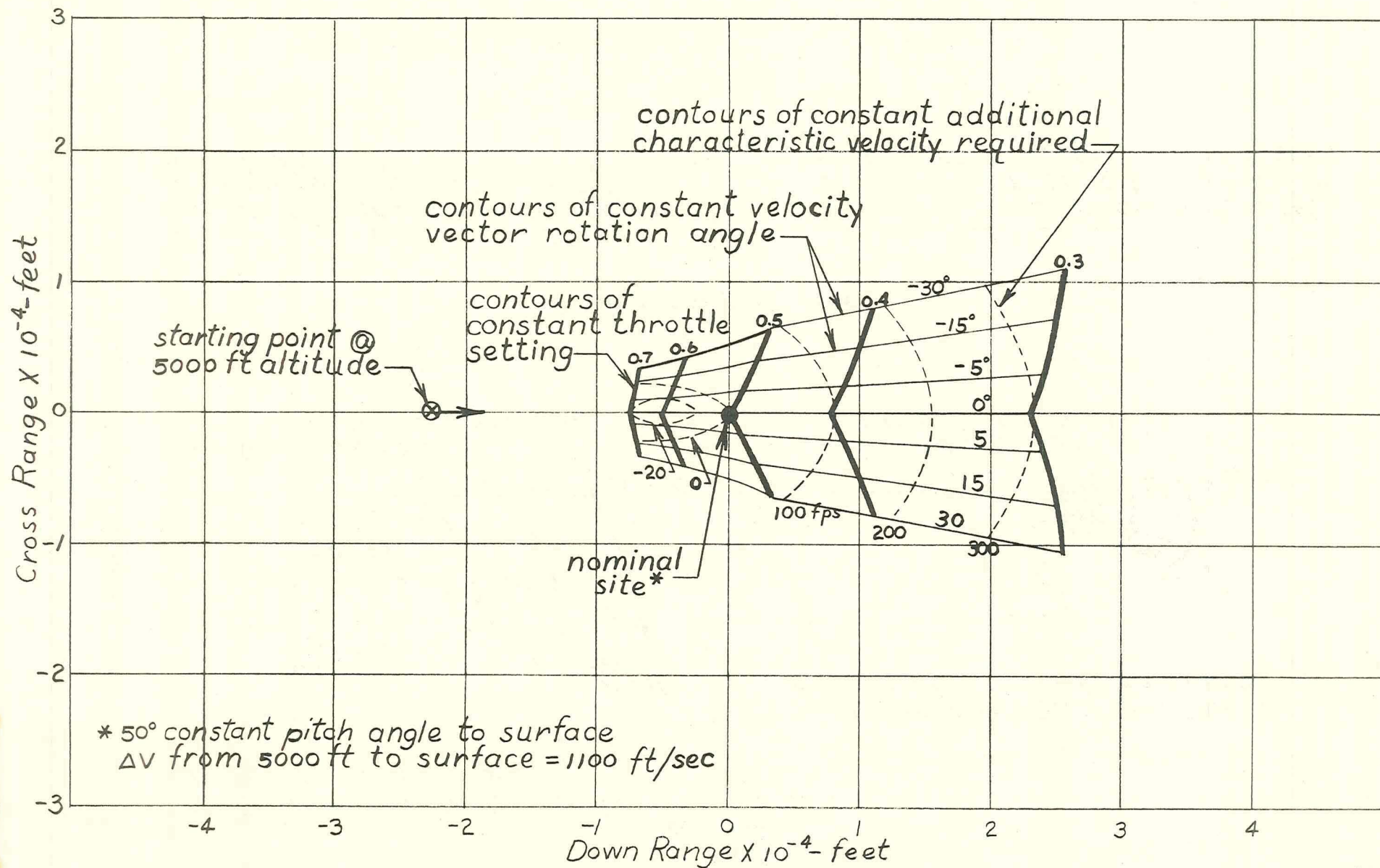


Figure 6.- Landing area footprint for corrections made at 5,000 feet altitude with velocity = 630 ft/sec and flight path angle of 12°.

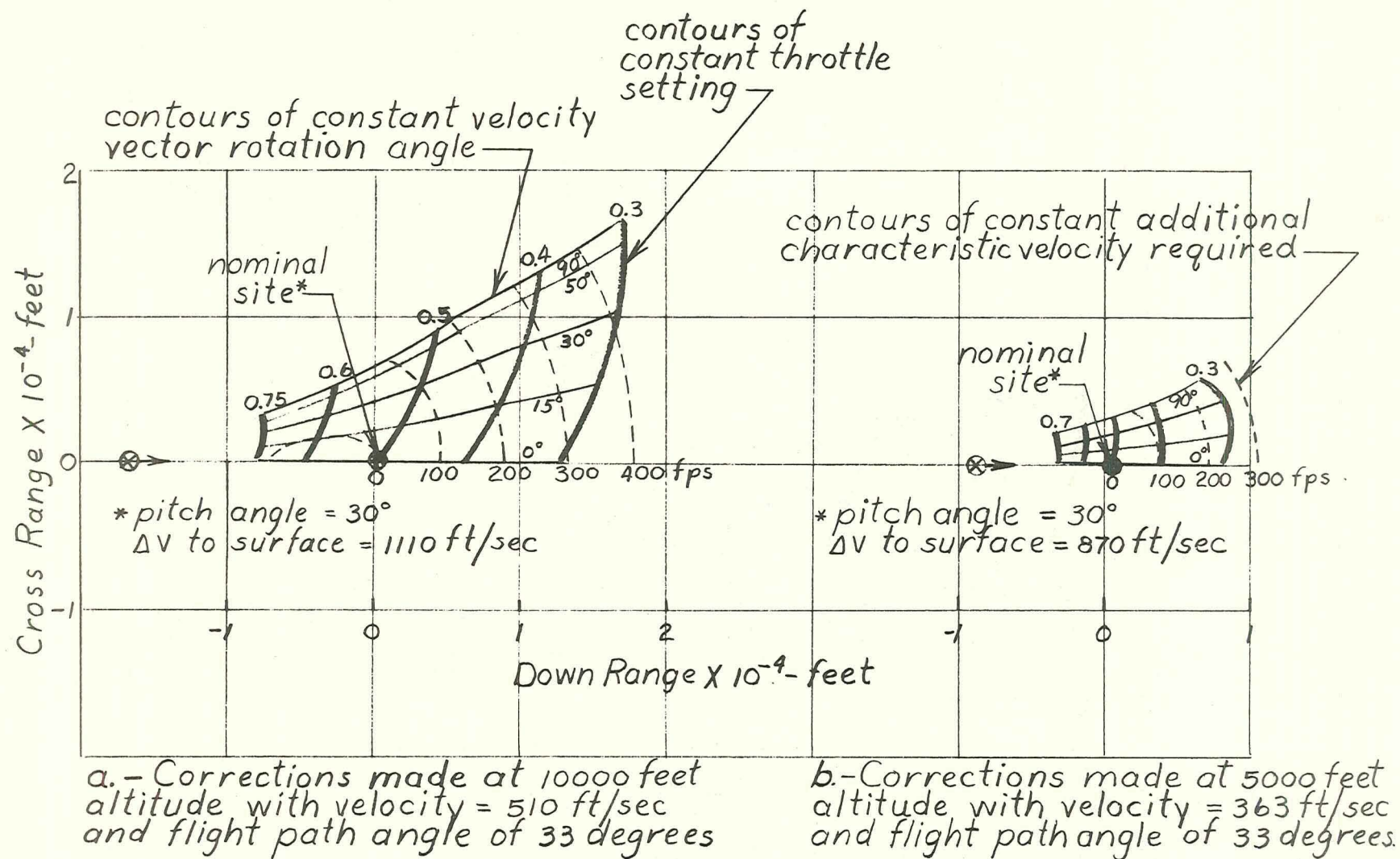


Figure 7.- Landing area half footprints for flight path angle of  $33^\circ$ .

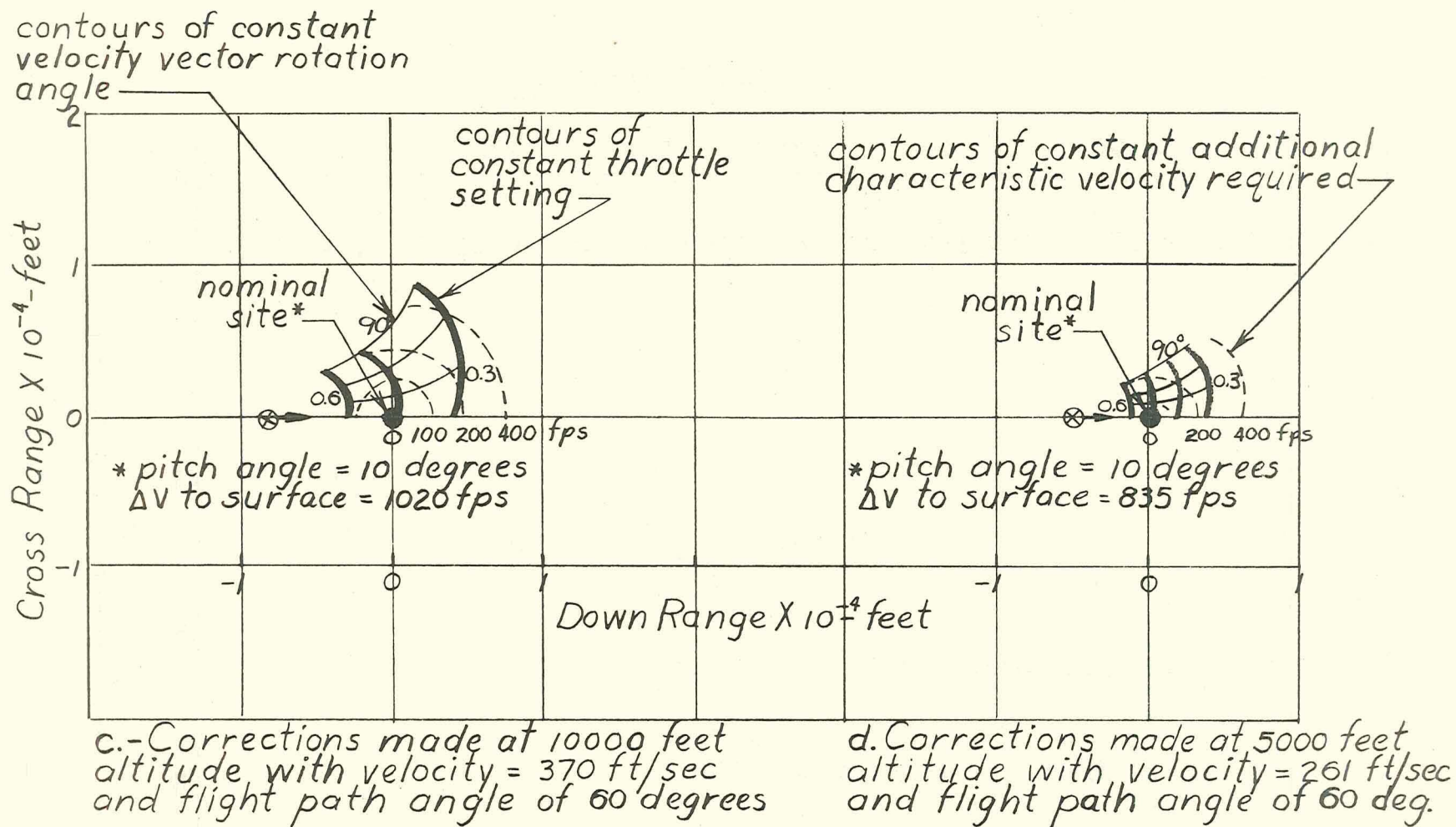


Figure 8.- Landing area half footprints for flight path angle of 60°.

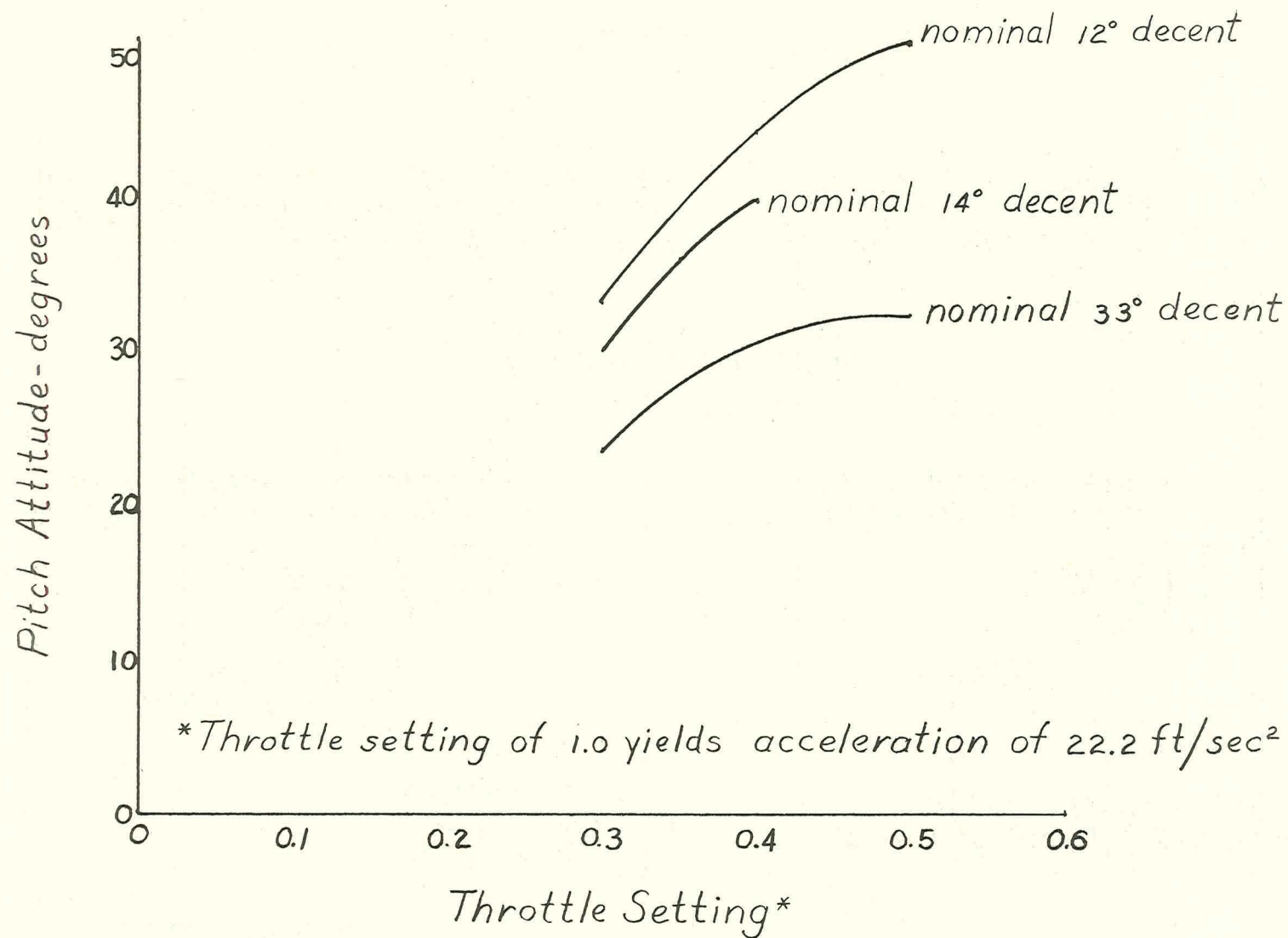


Figure 9.- Required pitch attitude (from any altitude) versus the required throttle setting for in plane landings beyond the nominal site.

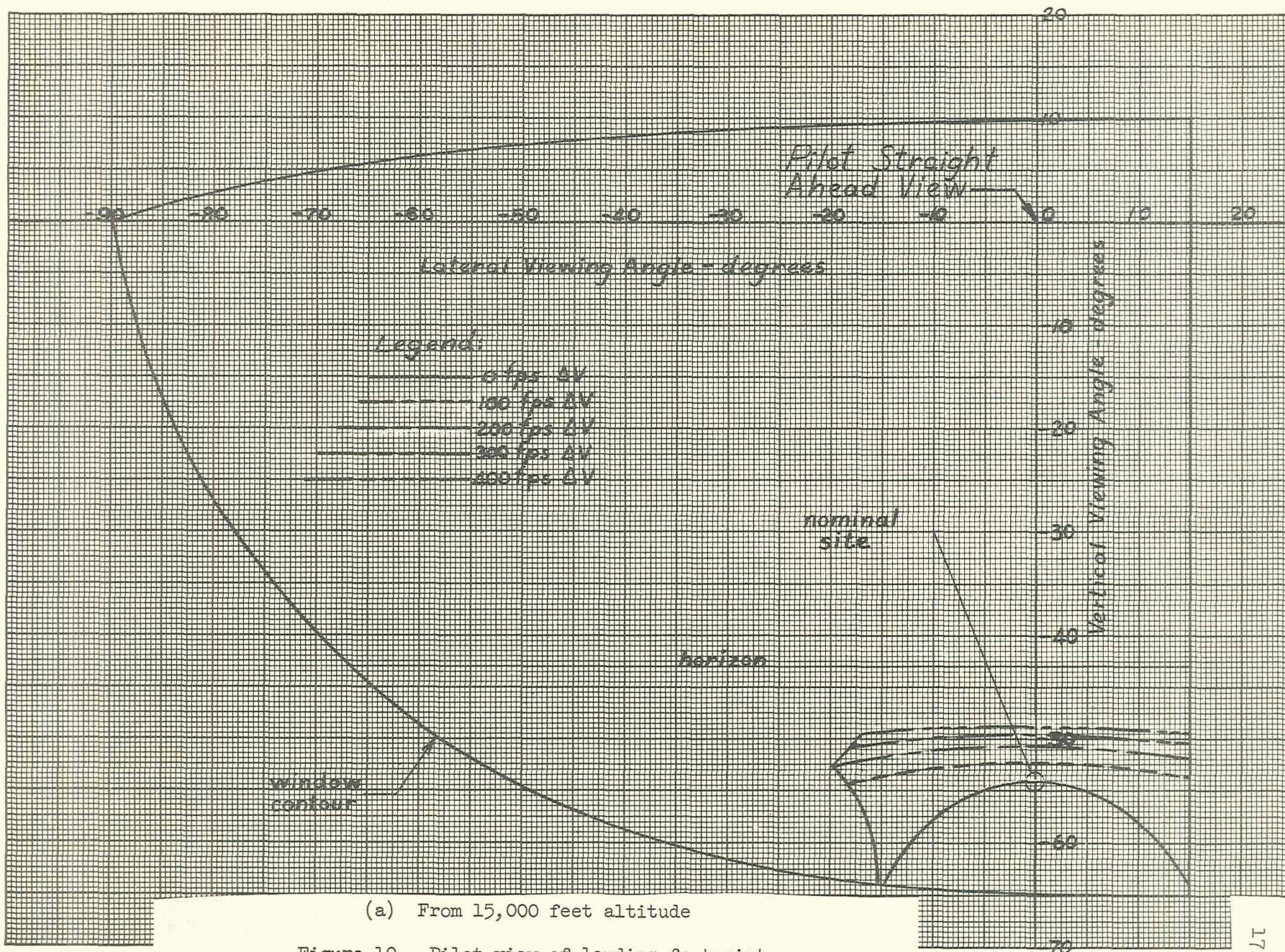


Figure 10.- Pilot view of landing footprint.

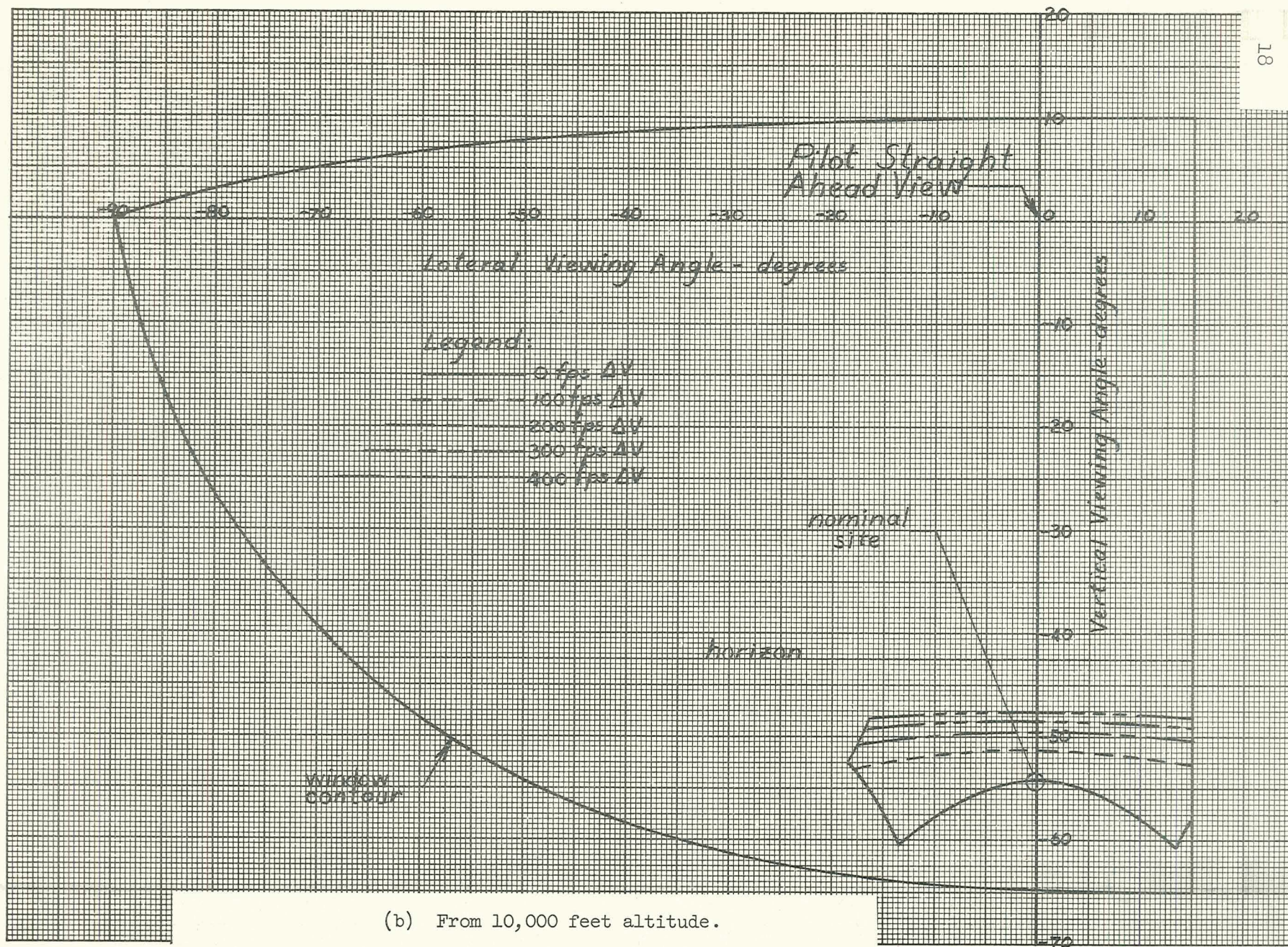


Figure 10.- Continued.

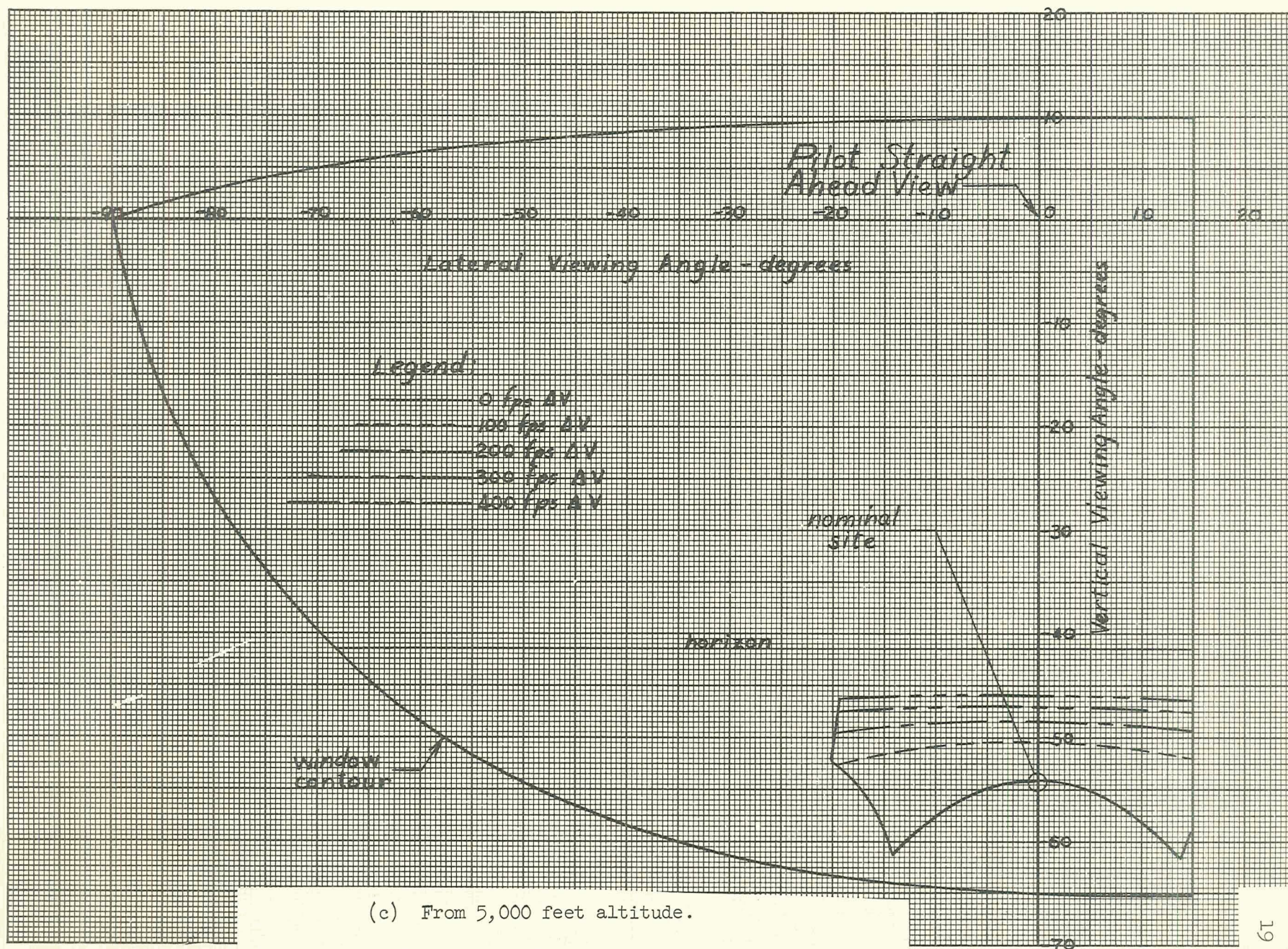


Figure 10.- Continued.

